Warm-up

Multiple choice questions from 3rd, 4th & 5th grade Released Test Questions, California Standards Tests

1

On Friday 1250 people visited the zoo. Three times as many people visited on Saturday than on Friday. How many people visited on Saturday?

A 3615

B 3650

C 3750

D 3753

- Prove your answer is correct using three different methods.

3 NS 2.4

2

Six students were sitting at each table in the lunch room. There are 34 tables. How many students were sitting in the lunch room?

A 208

B 204

C 188

D 1,824

- What errors would cause students to choose the incorrect answers?

3NS 2.4

3

There are 58 cases of soda in a warehouse. If there are 24 cans of soda in each case, how many cans of soda are in the warehouse?

A 1392

B 1362

C 1292

D 1262

- Use two different methods to prove your answer is correct.

4 NS 3.2, NS 3.3

4

Veronica can type 28 words per minute. At this rate, how many words can she type in 5.5 minutes?

A 154

B 157

C 159

D 162

- What diagrams or pictures could students draw to help them decide how to solve this problem? Draw 2 diagrams.

5 NS 2.1

Multiplying One-digit Numbers by Multi-Digit Numbers

Objective:

Teachers are given multiple strategies for teaching 3rd grade students to multiply one-digit numbers by multi-digit numbers. Students use of multiple strategies will foster understanding of the use of the distributive property of multiplication over addition, and when students are taught the standard algorithm they will have a solid understanding of the real math behind the procedures. Strategies used include the use of base-10 blocks, area models, the generic rectangle, expanded form, partial products, and bar models.

Standard:

3NS 2.4 - Solve simple problems involving multiplication of multi-digit numbers by one-digit numbers. (e.g. $3{,}671 \times 3 =$ _____).

Multiple Strategies for Side-by-Side comparisons:

- Base-10 blocks, with semi-concrete diagrams
- Area Models
- Bar Models and other diagrams
- Decomposition Methods:
    - Generic Rectangle
    - Partial Products with Expanded Form
- Partial Products

Multiplying One-Digit by Multi-Digit Numbers

Objective: Students will use multiple strategies to multiply multi-digit numbers by one-digit numbers.

Materials: Sets of base 10 blocks for pairs of students, including 100 flats, 10 rods, and 1 units, whiteboards and whiteboard markers, 1 each per student, paper and pencils for students to record work

Vocabulary: product, factor, multiply, combine equal groups, partial products, products

Prerequisite Lessons:

Single digit multiplication lessons and lessons on the concept of multiplication as repeated addition or combining of equal groups, as well as expanded form, addition of multi-digit numbers, and using base-10 blocks to represent multi-digit numbers.

Note: These lessons are best taught over multiple sessions, giving students the opportunity to become comfortable with each pair of strategies before teaching a new strategy.

Notes About the Teaching Cycle: Throughout these lessons, the teacher

- introduces the learning objective and connects to students' prior knowledge
- models new strategies with a common example, and makes connections to previous strategies.
- explicitly defines, uses and expects students to use vocabulary: (e.g. factors, products)
- provides structured Guided Practice, several problems the class does with the teacher
- assesses student learning during Guided Practice and with Student You Try problem
- Several Homework problems are provided for each group of strategies

During Guided Practice:

- Students work along with teacher to build, draw semi-concrete representations, and show work with equations on whiteboards or paper.
- Teacher monitors and provides corrective feedback.
- Teacher involves students throughout the guided practice using questions, and students respond with choral response and think, pair, share woven into instruction.
- Teacher calls on non-volunteers after partner sharing to check for understanding.

During Student You Tries:

- Students build, draw semi-concrete representations, and show work with equations.
- Teacher monitors and gives corrective feedback or additional guided practice and "you tries" as needed.

Bar Models and Base-10 Blocks:

Teacher presents problem, modeling while sharing a think-aloud with students:

- bar model and repeated addition
- building using base-10 blocks and combining with regrouping
- semi-concrete drawing of base-10 blocks
- multiplication equations

"There are 24 third graders in each of 3 classes. How many third graders are there altogether?"

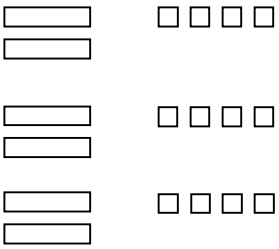
Bar Model:

24	24	24
?		

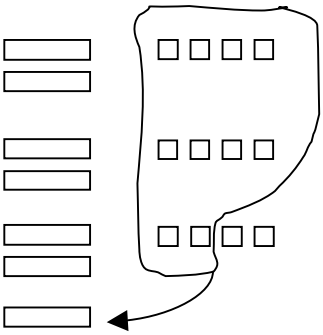
Repeated Addition:

24 + 24 + 24
= 20 + 4 + 20 + 4 + 20 + 4
= 20 + 20 + 20 + 4 + 4 + 4
= 60 + 12
= 72

Build it:



Show with regrouping:



6 tens and 12 ones

= 60 + 12
= 60 + 10 + 2
= 70 + 2
= 72

7 tens and 2 ones

= 70 + 2
= 72

Multiplication equation: $3 \times 24 = 72$

The product of 3 and 24 is 72. There are 72 third graders all together.

Note: Post the above example, including all parts, for students to refer to during this series of lessons.

Guided Practice:

Guided Practice #1:

"There are 4 ponds. Each pond has 14 frogs in it. What is the total number of frogs in the ponds?"

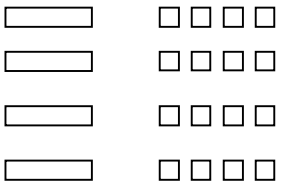
Bar Model:

?			
14	14	14	14

Repeated Addition:

$$\begin{aligned} & 14 + 14 + 14 + 14 \\ &= 10 + 4 + 10 + 4 + 10 + 4 + 10 + 4 \\ &= 10 + 10 + 10 + 10 + 4 + 4 + 4 + 4 \\ &= 40 + 16 \\ &= 40 + 10 + 6 \\ &= 50 + 6 \\ &= 56 \end{aligned}$$

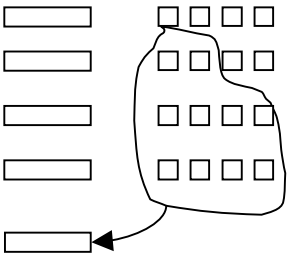
Build it:



4 tens and 16 ones

$$\begin{aligned} & 40 + 16 \\ &= 40 + 10 + 6 \\ &= 50 + 6 \\ &= 56 \end{aligned}$$

Show with regrouping:



5 tens and 6 ones

$$\begin{aligned} & 50 + 6 \\ &= 56 \end{aligned}$$

Multiplication Equation:

$4 \times 14 = 56$

The product of the factors 4 and 14 is 56. The total number of frogs is 56.

Guided Practice #2:

"There were 5 soccer fields at a park. Each field had 32 kids playing soccer. How many kids in all?"

Bar Model:

32	32	32	32	32
?				

Repeated Addition:

$32 + 32 + 32 + 32 + 32$

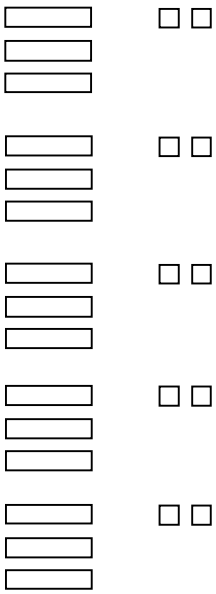
$= 30 + 30 + 30 + 30 + 30 + 2 + 2 + 2 + 2 + 2$

$= 90 + 60 + 10$

$= 100 + 60$

$= 160$

Build It:

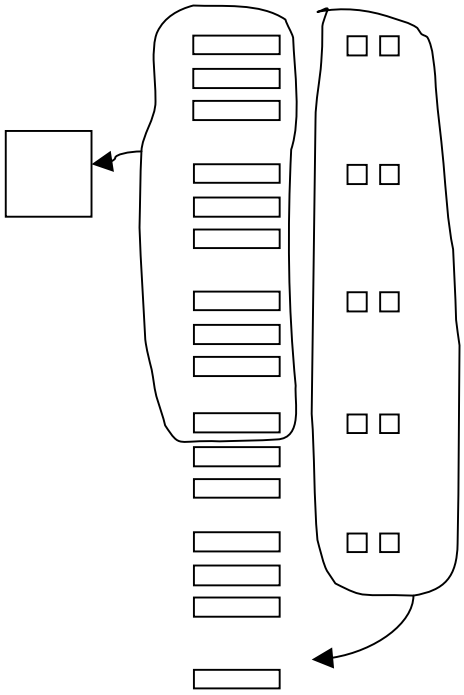


15 tens and 10 ones

$150 + 10$

$= 160$

Show with Regrouping:



1 hundred, 6 tens and 0 ones

$100 + 60$

$= 160$

Multiplication equation:

$5 \times 32 = 160$

The product of 5 and 32 is 160. There were 160 kids in all.

Student "You Try":

"Three friends each had 45 stickers. How many stickers did they have altogether?"

Bar Model:

45	45	45
?		

Repeated Addition:

$45 + 45 + 45$

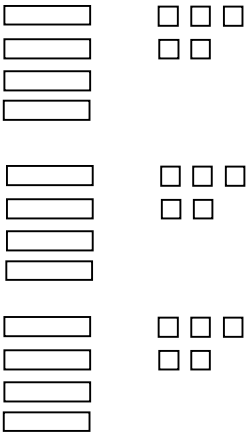
$= 40 + 40 + 40 + 5 + 5 + 5$

$= 80 + 20 + 20 + 10 + 5$

$= 100 + 30 + 5$

$= 135$

Build it:



12 tens and 15 ones

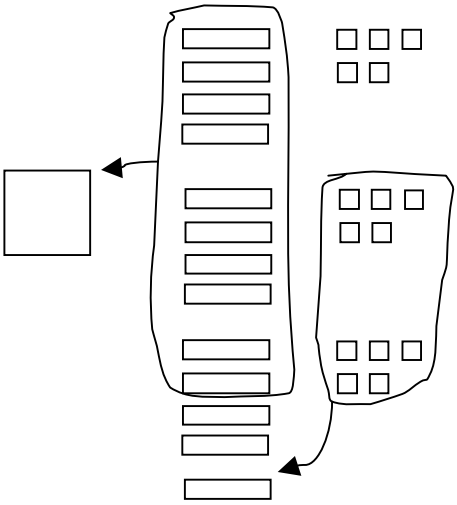
$120 + 15$

$= 120 + 10 + 5$

$= 130 + 5$

$= 135$

Show with regrouping:



1 hundred, 3 tens and 5 ones

$100 + 30 + 5$

$= 130 + 5$

$= 135$

Multiplication Equation: $3 \times 45 = 135$

The product of 3 and 45 is 135. They had 135 stickers altogether.

Closure: Use posted sample to summarize all the methods used to show multiplication so far. For homework students solve 2 problems using all the methods practiced in class. Homework keys attached.

1) $5 \times 35 =$ 2) 3×68

Area Model and Generic Rectangle:

Teacher:

- reviews posted work from previous lesson
- presents same original problem and demonstrates with a think-aloud, building an area model with base 10-blocks and drawing an area model to solve the problem
- presents the generic rectangle method, making connections with the area model

"There are 24 third graders in each of 3 classes. How many third graders are there altogether?"

Area Model:

		20		+		4	
		10	10	1	1	1	1
3	1	10	10	1	1	1	1
	1	10	10	1	1	1	1
	1	10	10	1	1	1	1

Equation:

$$\begin{aligned} & 3 \times 24 \\ &= 3 \times (20 + 4) \\ &= (3 \times 20) + (3 \times 4) \\ &= 60 + 12 \\ &= 72 \end{aligned}$$

Generic Rectangle:

Factors only:

	20	4
3		

With partial products:

	20	4
3	60	12

Sum of Products:

$$\begin{array}{r} 60 \\ +12 \\ \hline 72 \end{array}$$

Teacher compares and notes connections between the two models and posts these examples in addition to the previous methods for the same problem.

Note: As you continue with the guided practice make note if students need more explicit instruction and practice with:

- multiplying times 10s, 100s, and 1000s
- expanded form

Guided Practice:

Guided Practice #1:

"There are 4 ponds. Each pond has 14 frogs in it. What is the total number of frogs in the ponds?"

Area Model:

		10	1	1	1	1
		10	+	4		
4	1	10	1	1	1	1
	1	10	1	1	1	1
	1	10	1	1	1	1
	1	10	1	1	1	1

Generic Rectangle:

	10	+	4
4	40		16

Sum:

$$\begin{array}{r} 40 \\ +16 \\ \hline 56 \end{array}$$

Equation:

$$\begin{aligned} & 4 \times 14 \\ &= 4 \times (10 + 4) \\ &= (4 \times 10) + (4 \times 4) \\ &= 40 + 16 \\ &= 56 \end{aligned}$$

The product of 4 and 14 is 56. There is a total of 56 frogs in the 4 ponds.

Guided Practice #2:

"There were 5 soccer fields at a park. Each field had 32 kids playing soccer. How many kids were playing soccer altogether?"

Area Model:

		10	10	10	1	1
		30			+	2
5	1	10	10	10	1	1
	1	10	10	10	1	1
	1	10	10	10	1	1
	1	10	10	10	1	1
	1	10	10	10	1	1

Generic Rectangle:

	30	+	2
5	150		10

Sum: $150 + 10 = 160$

Equation:

$$\begin{aligned} & 5 \times 32 \\ &= 5 \times (30 + 2) \\ &= (5 \times 30) + (5 \times 2) \\ &= 150 + 10 \\ &= 160 \end{aligned}$$

The product of 5 and 32 is 160.

There are 160 kids playing soccer altogether.

Student "You Try":

"Five friends each had 45 stickers. How many stickers did they have in all?"

Area Model:

		40				+	5			
		10	10	10	10	1	1	1	1	1
	1	10	10	10	10	1	1	1	1	1
	1	10	10	10	10	1	1	1	1	1
5	1	10	10	10	10	1	1	1	1	1
	1	10	10	10	10	1	1	1	1	1
	1	10	10	10	10	1	1	1	1	1

Generic Rectangle:

	40	+	5
5	200		25

Sum: $200 + 25$

$= 225$

Equation:

5×45

$= 5 \times (40 + 5)$

$= (5 \times 40) + (5 \times 5)$

$= 200 + 25$

$= 200 + 25$

$= 225$

The product of 5 and 45 is 225. They had 225 stickers in all.

Closure: Use posted sample to summarize all the methods used to show multiplication today. For homework students solve 2 problems using methods practiced in class. Homework keys attached.

1) $4 \times 13 =$ 2) 2×34

Partial Products with Expanded Form and Partial Products

Teacher:

- reviews posted work from previous lessons with class
- presents same original problem and demonstrates with a think-aloud, modeling partial products with expanded form and partial products methods
- leads class in making connections across all methods used so far

"There are 24 third graders in each of 3 classes. How many third graders are there altogether?"

Partial Products with Expanded Form:

$$\begin{array}{rll} 24 & = 20 + 4 & \\ \times\ 3 & \times\quad 3 & \\ & 12 & = 3 \times 4 \\ & +\ 60 & = 3 \times 20 \\ \hline & 72 & \end{array}$$

Partial Products:

$$\begin{array}{rl} 24 & \\ \times\quad 3 & \\ 12 & = 3 \times 4 \\ +\ 60 & = 3 \times 20 \\ \hline 72 & \end{array}$$

We showed with two more ways that the product of 3 and 24 is 72.

Note: Post these additional methods with previous posters.

Teacher:

- Make connections using highlighters on posters: factors, partial products, products
- Vocabulary Practice - Provide Language frames for students, especially students who are English Learners, such as:

  "____ and ____ are _____. I know _____ because _____."

  Sample response: "3 and 24 are factors. I know 3 is a factor because factors are numbers you multiply together, and we multiplied 3 times 4."
- Students turn to partners to practice responses prior to sharing with class.

Note: As you continue with the guided practice make note if students need more explicit instruction and practice with:

- multiplying times 10s, 100s, and 1000s
- expanded form

Guided Practice:

Guided Practice #1:

"There are 4 ponds. Each pond has 14 frogs in it. What is the total number of frogs in the ponds?"

Partial Products with Expanded Form:

$$
\begin{array}{rrl}
14 & = 10 + 4 & \\
\times 4 & \times \quad 4 & \\
\hline
 & 16 & = 4 \times 4 \\
 & + \quad 40 & = 4 \times 10 \\
\hline
 & 56 &
\end{array}
$$

Partial Products:

$$
\begin{array}{rrl}
 & 14 & \\
\times & 4 & \\
\hline
 & 16 & = 4 \times 4 \\
+ & 40 & = 4 \times 10 \\
\hline
 & 56 &
\end{array}
$$

The product of 4 and 14 is 56. The total number of frogs is 56.

Guided Practice #2:

"Three schools have 135 third graders each. What is the total of all the 3rd graders?

Partial Products with Expanded Form:

$$
\begin{array}{rrl}
135 & = 100 + 30 + 5 & \\
\times 3 & \times \quad 3 & \\
\hline
 & 15 & = 3 \times 5 \\
 & 90 & = 3 \times 30 \\
 & + \quad 300 & = 3 \times 100 \\
\hline
 & 405 &
\end{array}
$$

Partial Products:

$$
\begin{array}{rrl}
 & 135 & \\
\times & 3 & \\
\hline
 & 15 & = 3 \times 5 \\
 & 90 & = 3 \times 30 \\
+ & 300 & = 3 \times 100 \\
\hline
 & 405 &
\end{array}
$$

We showed that the product of 3 and 135 is 405. The total number of 3rd graders is 405.

Student "You Try":

You Try:

"Two schools each had 346 students. How many students attended the schools in all?"

Partial Products with Expanded Form:

$$
\begin{array}{rrl}
346 & = 300 + 40 + 6 & \\
\times 2 & \times \quad 2 & \\
\hline
 & 12 & = 2 \times 6 \\
 & 80 & = 2 \times 40 \\
 & + \quad 600 & = 2 \times 300 \\
\hline
 & 692 &
\end{array}
$$

Partial Products:

$$
\begin{array}{rrl}
 & 346 & \\
 & \times \quad 2 & \\
\hline
 & 12 & = 2 \times 6 \\
 & 80 & = 2 \times 40 \\
 & + \quad 600 & = 2 \times 300 \\
\hline
 & 692 &
\end{array}
$$

We showed that the product of 2 and 346 is 692. 692 students attend the schools in all.

*Note: Writing the multiplication equations of the factors next to the partial products is a step to use when first using these strategies, but once students understand how to solve for the partial products, they don't have to keep writing the equations alongside.

Closure: Now we've shown quite a few different ways to show how we multiply a one-digit factor by a multi-digit factor to solve for a product. Review the posters with the students.

For homework, use the partial products with expanded form, and the partial products methods to show your work on these three problems:

1) $5 \times 42 =$ 2) $3 \times 307 =$ 3) $5 \times 741 =$

Further Notes and Extensions:

Standard Algorithm - The standard multiplication algorithm should be presented following these presentations. Students should be encouraged to show their work with multiple methods, including the standard algorithm, even after they've learned the standard method. Students will eventually choose a method that works best for them.

Homework - Once all the strategies have been taught and practiced, students can be asked to solve a small number of problems and choose two or three methods to show their work for each problem.

Facts Practice - Since 3rd grade students are still learning their multiplication facts, this is a chance for them to practice learning them through using them. Some teachers allow students to use their facts tables when learning the algorithms. Students also need to be explicitly taught to use a variety of strategies for figuring out unknown facts while solving a problem, (e.g. draw equal groups, repeated addition, use a known fact, skip counting, number lines...)

Varied problems - As students progress and continue practicing problems, be sure to mix in problems with zeros, problems that don't always require regrouping, problems to thousands...

Homework note: Teacher can differentiate by assigning different numbers of problems to different students. As students become proficient at the different methods, the teacher can also differentiate by directing students to choose to do the homework problems with any method they choose, making sure they use each method at least once, (the generic rectangle, base-10 blocks, and/or bar models, partial products with expanded form, partial products, or the traditional method).

Side-by-Side Comparison of Multiple Strategies - $3 \times 24 = 72$

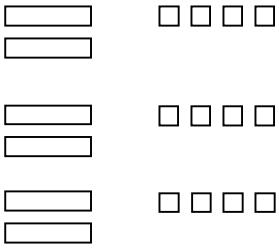
Bar Model with Repeated Addition

24	24	24
?		

$$\begin{aligned} & 24 + 24 + 24 \\ &= 20 + 4 + 20 + 4 + 20 + 4 \\ &= 20 + 20 + 20 + 4 + 4 + 4 \\ &= 60 + 12 \\ &= 72 \end{aligned}$$

Base-10 Blocks

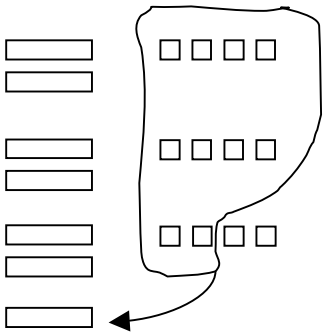
Build It



6 tens and 12 ones

$$\begin{aligned} &= 60 + 12 \\ &= 60 + 10 + 2 \\ &= 70 + 2 \\ &= 72 \end{aligned}$$

With Regrouping:



7 tens and 2 ones

$$\begin{aligned} &= 70 + 2 \\ &= 72 \end{aligned}$$

Area Model:

	20		+	4	
	10	10	1	1	1	1
1	10	10	1	1	1	1
1	10	10	1	1	1	1
1	10	10	1	1	1	1

3

Equation:

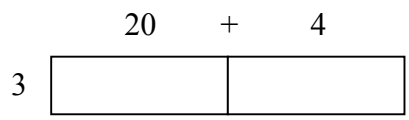
$$\begin{aligned} & 3 \times 24 \\ &= 3 \times (20 + 4) \\ &= (3 \times 20) + (3 \times 4) \\ &= 60 + 12 \\ &= 72 \end{aligned}$$

(Continued)

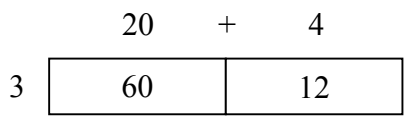
Side-by-Side Comparison Continued:

Generic Rectangle:

Factors only:



With partial products:



Sum:

$$\begin{array}{r} 60 \\ +12 \\ \hline 72 \end{array}$$

Partial Products with Expanded Form:

$$\begin{array}{rll} 24 & = 20 + 4 & \\ \times\, 3 & \times \quad 3 & \\ \hline & 12 & = 3 \times 4 \\ & + \quad 60 & = 3 \times 20 \\ \hline & 72 & \end{array}$$

Partial Products:

$$\begin{array}{rll} & 24 & \\ \times & 3 & \\ \hline & 12 & = 3 \times 4 \\ + & 60 & = 3 \times 20 \\ \hline & 72 & \end{array}$$

Homework Key - Bar Models and Base-10 Blocks

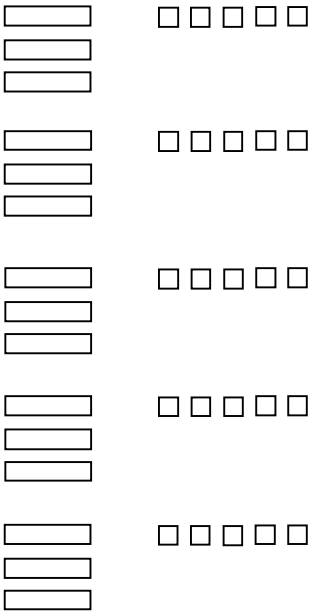
1) $5 \times 35 = 175$

Bar Model with Repeated Addition:

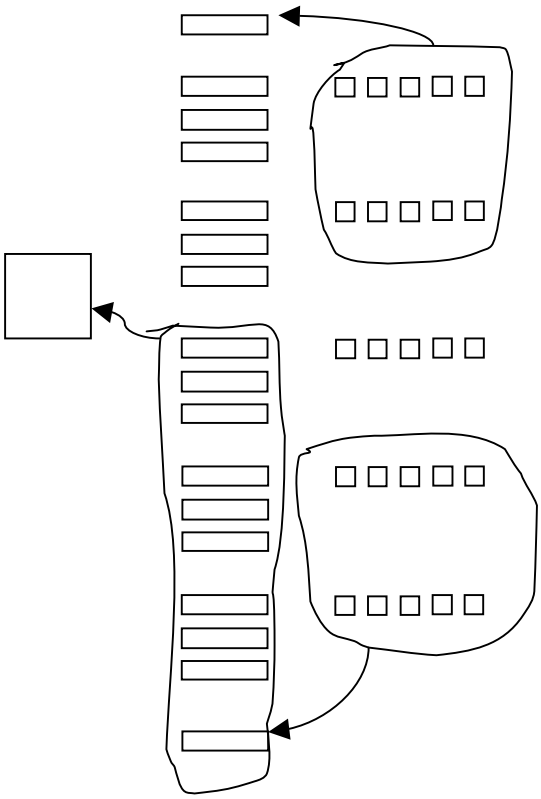
35	35	35	35	35
?				

$35 + 35 + 35 + 35 + 35$
$= 30 + 30 + 30 + +30 + 30 + 5 + 5 + 5 + 5 + 5$
$= 90 + 60 + 25$
$= 90 + 10 + 50 + 25$
$= 100 + 75$
$= 175$

Build it:



Show with regrouping:



15 tens and 25 ones
$= 150 + 25$
$= 175$

1 hundred, 7 tens and 5 ones
$= 100 + 70 + 5$
$= 175$

Homework Key - Bar Models and Base-10 Blocks

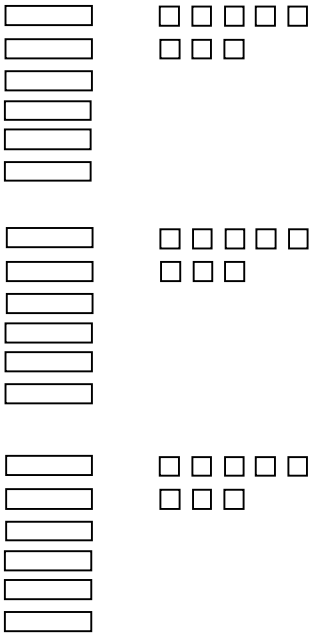
2) $3 \times 68 = 204$

Bar Model with Repeated Addition:

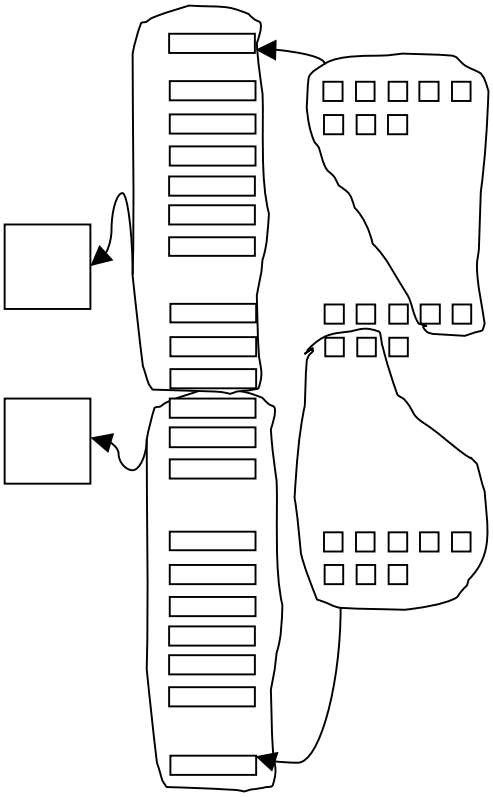
68	68	68
?		

$68 + 68 + 68$
$= 60 + 60 + 60 + 8 + 8 + 8$
$= 180 + 24$
$= 180 + 20 + 4$
$= 200 + 4$
$= 204$

Build it:



Show with regrouping:



18 tens and 24 ones
$= 180 + 24$
$= 180 + 20 + 4$
$= 200 + 4$
$= 204$

2 hundreds and 4 ones
$= 200 + 4$
$= 204$

Homework Key - Area Models and Generic Rectangles

1) $4 \times 13 = 52$

Area Model:

		10	1	1	1
	1	10	1	1	1
4	1	10	1	1	1
	1	10	1	1	1
	1	10	1	1	1

$$\begin{aligned} & 4 \times 13 \\ &= 4 \times (10 + 3) \\ &= (4 \times 10) + (4 \times 3) \\ &= 40 + 12 \\ &= 40 + 10 + 2 \\ &= 50 + 2 \\ &= 52 \end{aligned}$$

Generic Rectangle:

	10	3
4	40	12

Sum of the Partial Products:

$$\begin{array}{r} 40 \\ +12 \\ \hline 52 \end{array}$$

2) $2 \times 34 = 68$

Area Model:

		10	10	10	1	1	1	1
2	1	10	10	10	1	1	1	1
	1	10	10	10	1	1	1	1

$$\begin{aligned} & 2 \times 34 \\ &= 2 \times (30 + 4) \\ &= (2 \times 30) + (2 \times 4) \\ &= 60 + 8 \\ &= 68 \end{aligned}$$

Generic Rectangle:

	30	4
2	60	8

Sum of the Partial Products:

$$\begin{array}{r} 60 \\ +\ 8 \\ \hline 68 \end{array}$$

Homework Key - Partial Products with Expanded Form and Without Expanded Form

1) $5 \times 42 = 210$

Partial Products with Expanded Form:

$$\begin{array}{rll} 42 & = 40 + 2 & \\ \times 5 & \times \quad 5 & \\ & 10 & = 5 \times 2 \\ & + \quad 200 & = 5 \times 40 \\ & 210 & \end{array}$$

Partial Products:

$$\begin{array}{rl} 42 & \\ \times \quad 5 & \\ 10 & = 5 \times 2 \\ + \quad 200 & = 5 \times 40 \\ 210 & \end{array}$$

2) $3 \times 307 = 921$

Partial Products with Expanded Form:

$$\begin{array}{rll} 307 & = 300 + 7 & \\ \times \ 3 & \times \quad 3 & \\ & 21 & = 3 \times 7 \\ & + \quad 900 & = 3 \times 300 \\ & 921 & \end{array}$$

Partial Products:

$$\begin{array}{rl} 307 & \\ \times \ 3 & \\ 21 & = 3 \times 7 \\ + 900 & = 3 \times 300 \\ 921 & \end{array}$$

3) $5 \times 741 = 3705$

Partial Products with Expanded Form:

$$\begin{array}{rlll} 741 & = 700 + 40 + 1 & & \\ \times \ 5 & \times \quad 5 & & \\ & 5 & = 5 \times 1 & \\ & 200 & = 5 \times 40 & \\ & + \ 3500 & = 5 \times 700 & \\ & 3705 & & \end{array}$$

Partial Products:

$$\begin{array}{rl} 741 & \\ \times \quad 5 & \\ 5 & = 5 \times 1 \\ 200 & = 5 \times 40 \\ + \ 3500 & = 5 \times 700 \\ 3705 & \end{array}$$